

Focus
Standards
and Claim

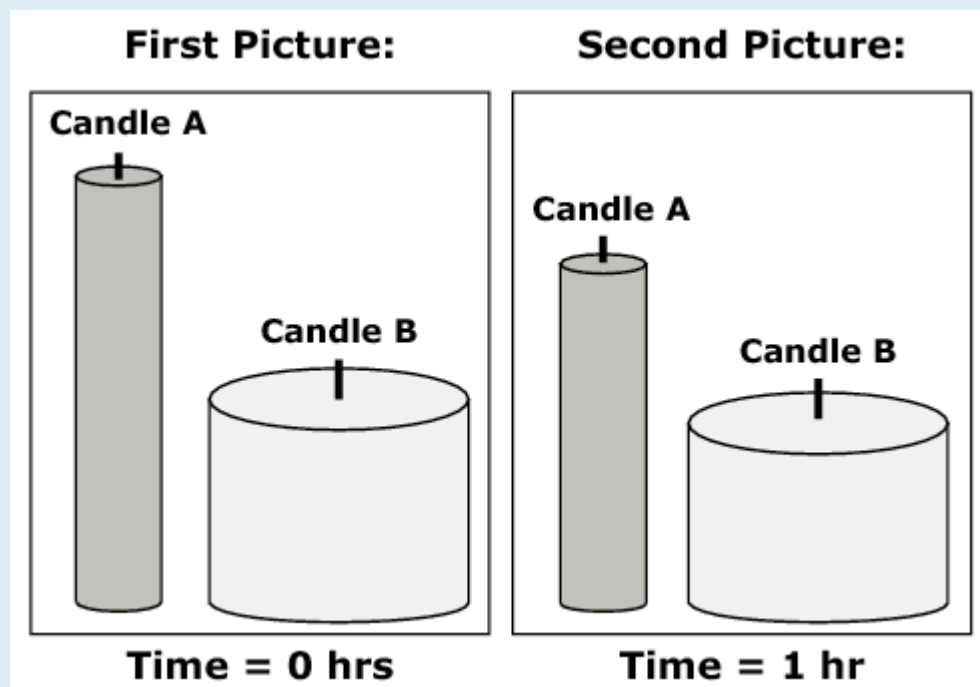
Claim 2
8.EE.C.8

Stimulus

Lights, Candles, Action!

Your friend Abbie is making a movie. She is filming a fancy dinner scene and she has two types of candles on the table. She wants to determine how long the candles will last.

She takes a picture, lights the candles, and then lets them burn for 1 hour. She then takes a second picture. You can assume that each candle burns at its own constant rate.



Candle Type A initial height = 20 cm

Candle Type B initial height = 10 cm

Candle Type A height after burning for 1 hour = 16 cm

Candle Type B height after burning for 1 hour = 9 cm

You will use this information to help Abbie think about the candles she might use for her film.

<h3 style="margin: 0;">Item Prompt</h3>	<p>Abbie has 3 hours left to film. She lights a new Candle Type A and Candle Type B and then starts filming.</p> <p>In the 3 hours she has left, will Abbie capture the moment when the candles are exactly the same height?</p> <p>Explain to Abbie how you can determine the answer.</p>
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Scoring Guide

SCORE	2 POINTS	1 POINT	0 POINTS
	<p>The student correctly answers No, Abbie will not capture the moment when Candle Type A and Candle Type B are the same height during the 3 hours of filming, AND supports the claim with a mathematically valid argument.</p> <p>Note: The underlying content is about Systems of Linear Equations; however, students are not required to set up, symbolically, a system of linear equations in order to solve this problem.</p>	<p>Student correctly answers No, Abbie will not capture the moment when Candle Type A and Candle Type B are the same height during the 3 hours of filming, but does not support the claim with a mathematically valid argument.</p> <p>OR</p> <p>Student correctly reasons from an incorrect calculation.</p>	<p>All other responses.</p>

Sample Responses

Student Sample A



$X = \text{hours}$

$$A(h) = 20 - 4x$$

$$B(h) = 10 - x$$

$$20 - 4x = 10 - x$$

$$10 = 3x$$

$$X = 10/3 \text{ hour}$$

$$X = 3 \frac{1}{3}$$

No Abbey will not be able to capture the moment where the candles are the same height because equations that represent the decreasing height can be constructed for A and B. When those equations are set equal to each other, it represents when at what time the height of candles are equal. Solving that equation, x is found equal to $3 \frac{1}{3}$ hour which is past 3 hours. Also a table with the height of both A and B can be constructed.

Hour	1	2	3
Candle A	16	12	8
Candle B	9	8	7

SCORE RATIONALE

The student developed a valid mathematical argument to support the claim that Candle Type A and Candle Type B will not reach the same height within the 3-hour time frame. The student also wrote and solved a system of two linear equations to show that both candles will have the same height at exactly $3 \frac{1}{3}$ hours after being lit, and provided a table of values to further support the argument. The response earns full credit.

Student Sample B



$X = \#$ of hours

$X =$ final height (cm)

$$-4x + 20 = -x + 10$$

$$10 = 3x$$

$$10/3 = x$$

You first create two equations, one for Candle A and one for Candle B. You then make them equal to each other, therefore making you solve for x . After finding x , you will find out that it will take around 3.3 hours to capture the moment when the candles are the same height. But with the time constraint of 3 hours, she won't be able to see the moment.

SCORE RATIONALE

The student developed and solved a system of linear equations to show that it will take "around 3.3 hours," which is beyond the 3-hour time limit, to capture the moment when both candles are the same height. The response also includes a clear and valid mathematical argument that connects the solution to the system of equations to the context. This response earns full credit.

Student Sample C



1 hour:

$$20 - 4 = 16$$

$$10 - 1 = 9$$

2 hours:

$$16 - 4 = 12$$

$$9 - 1 = 8$$

3 hours:

$$12 - 4 = 8$$

$$8 - 1 = 7$$

No, the candles won't be exactly the same height. Candle type A will be 8 cm while candle type B will be 7cm. Take the initial height subtract 4 cm (candle type A) or 1 cm (candle type B) for each hour that passes.

SCORE RATIONALE

The student presented calculations of the change in height of each candle for each hour of burning. These calculations are organized clearly to connect with the valid mathematical reasoning presented in the concluding argument. Although the response does not include setting up or solving a system of linear equations, this is not a requirement for full credit. The reasoning presented is clear, and the argument provided is valid and directly supports the claim that the candles will not be the same height within the 3-hour time frame. The response earns full credit.

Student Sample D



Abbie can determine her answer by remodifying the equations I used.

H = total height of the candle

O_a = Original Height of Candle Type A

O_b = Original Height of Candle Type B

t = hours spent burning

n = difference of height lost in 1 hour of burning

n_1 = Candle A

n_2 = Candle B

$H = O_a - n_1t$

$H = O_b - n_2t$

By using these equations, Abbie can determine if Candle Type A and Candle Type B will be the exact same height by determining the candles' height after a # of hours burning.

SCORE RATIONALE

The student identified a thorough set of variables to consider in the process of determining if both candles can be the same height in 3 hours. The student also wrote clear equations for the candles with these variables, positioned to correctly represent the relationships among the variables. The explanation suggests the student understood that solving the equations simultaneously would solve the problem. However, the response does not indicate if there will be time to capture the desired moment, nor does it include an argument. The response earns partial credit for providing evidence of reasoning correctly about a valid approach to solving the problem.

Student Sample E



Type A = 5 hours

Type B = 10 hours

Type B $7 - 3 = 4$ cm

Type A after 3hr = 8cm

No, Abbie will not capture the moment when the candles are exactly the same because after she burns 3 hrs before she only have 7cm left. Then after she burns another 3 hrs she will only have 4cm left. With Candle Type A after burning 3 hrs it will go down to 8cm because every hour it will burns off 4cm. So the height of both Type A & B are different by 2 times.

SCORE RATIONALE

The student correctly determined that both candles will not reach the same height within 3 hours. However, the response provides insufficient evidence of valid mathematical reasoning and includes a calculation error in the attempt to support the claim. The final statement of the response is hard to interpret. The response earns 1 point.

Student Sample F



In three hours, she will see the candles be about the same height. Since type A starts out at 20cm & type B starts out at 10cm, in three hours, type A would go down 12cm and type B would go out 3cm. It would be 8cm for A & 7cm for B.

SCORE RATIONALE

This response includes a reasonable attempt to construct a mathematical argument to support the claim that the candles will be about the same height in 3 hours. However, the explanation does not clearly indicate the burn rate of each candle, and is not precise enough to constitute a valid argument. The response earns 1 point.

Student Sample G



Yes it is possible, because since candle A loses 4cm in 1 hour and Candle B only loses 1cm in 1 hour then around the third hour they should be the same height. She would have to subtract 4cm from the height of candle A and subtract 1cm from the height of candle B until they reach the same height, but she can only subtract them 3 times or else she'll exceed her 3 hour goal.

SCORE RATIONALE

The student described the situation in a way that is approximately accurate, but not accurate enough to reach a correct conclusion about what will happen within 3 hours. There is partial evidence of an attempt to support the initial (incorrect) claim, but the response does not include a valid mathematical argument. The response earns 0 points.

Student Sample H



Yes, maybe.

Because candle A burns faster than candle b, candle A is just 1cm off of candle B, so at a point in the 3 hours, they will have the same height.

SCORE RATIONALE

After presenting some initial uncertainty, the student claimed that within 3 hours, the candles will have the same height. However, the response does not show enough mathematical work to support this incorrect claim, and there is not a clear argument. The response earns 0 points.

Student Sample I



According to the given evidence, candle A & B will both burn out before the new set of candles can match because the constant rate will make them decrease over time and the old candles have had more time to burn so they will run out quicker than the new ones.

SCORE RATIONALE

This response suggests that the student may have misunderstood the situation and may have been confused about what is asked in the prompt. There is evidence of some reasoning about the two candles, but a reference to "the constant rate" suggests that the student interpreted the candles to have the same constant rate. There also seems to be an unfortunate issue in understanding the meaning of "new set of candles" (versus "old candles"). The response earns 0 points.