Picking a Pet

Your class is trying to decide what type of animal to get for the class pet. Your teacher is letting the class vote to choose a goldfish, a turtle, or a hamster as the class pet.

All 20 students in your class voted for both their 1st choice and their 2nd choice for the class pet. The results are shown in Table 1.

Table 1. Class Pet Votes

<table>
<thead>
<tr>
<th>Student</th>
<th>1st Choice</th>
<th>2nd Choice</th>
<th>Student</th>
<th>1st Choice</th>
<th>2nd Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Turtle</td>
<td>Hamster</td>
<td>11</td>
<td>Turtle</td>
<td>Hamster</td>
</tr>
<tr>
<td>2</td>
<td>Goldfish</td>
<td>Hamster</td>
<td>12</td>
<td>Turtle</td>
<td>Goldfish</td>
</tr>
<tr>
<td>3</td>
<td>Goldfish</td>
<td>Turtle</td>
<td>13</td>
<td>Hamster</td>
<td>Turtle</td>
</tr>
<tr>
<td>4</td>
<td>Hamster</td>
<td>Turtle</td>
<td>14</td>
<td>Hamster</td>
<td>Goldfish</td>
</tr>
<tr>
<td>5</td>
<td>Goldfish</td>
<td>Turtle</td>
<td>15</td>
<td>Turtle</td>
<td>Goldfish</td>
</tr>
<tr>
<td>6</td>
<td>Turtle</td>
<td>Goldfish</td>
<td>16</td>
<td>Goldfish</td>
<td>Turtle</td>
</tr>
<tr>
<td>7</td>
<td>Hamster</td>
<td>Goldfish</td>
<td>17</td>
<td>Turtle</td>
<td>Goldfish</td>
</tr>
<tr>
<td>8</td>
<td>Turtle</td>
<td>Goldfish</td>
<td>18</td>
<td>Turtle</td>
<td>Goldfish</td>
</tr>
<tr>
<td>9</td>
<td>Goldfish</td>
<td>Hamster</td>
<td>19</td>
<td>Turtle</td>
<td>Hamster</td>
</tr>
<tr>
<td>10</td>
<td>Goldfish</td>
<td>Hamster</td>
<td>20</td>
<td>Goldfish</td>
<td>Hamster</td>
</tr>
</tbody>
</table>
Understanding

Item Prompt

Your teacher wants to use a point system to select the winning pet. She wants each pet to get a certain number of points for each 1st choice vote and a certain number of points for each 2nd choice vote.

Your teacher decides to use these rules for her point system:

• Points need to be positive whole numbers
• Points for a 1st choice vote have to be greater than or equal to the points for a 2nd choice vote.

Determine point values for the 1st and 2nd choice that would result in the turtle winning. Use words and numbers to explain how this point system results in the turtle winning.

Scoring Guide

<table>
<thead>
<tr>
<th>SCORE</th>
<th>2 POINTS</th>
<th>1 POINT</th>
<th>0 POINTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The student describes a method that meets the two criteria (assigns positive whole number of points to each 1st choice and 2nd choice, and 1st choice is greater or equal to the 2nd choice values) AND gives a valid explanation for why the turtle wins using this method.</td>
<td>The student describes a method that meets the two criteria (assigns positive whole number of points to each 1st choice and 2nd choice, and 1st choice is greater or equal to the 2nd choice values), but does not give a valid explanation for why the turtle wins using this method. OR The student gives a valid explanation for why the turtle wins using this method, but does not describe the method.</td>
<td>No viable method or viable explanation is provided.</td>
</tr>
</tbody>
</table>
Sample Responses

Student Sample A

If you had two points for every first choice and one point for every second place, the turtle would end up with 23, the hamster 15, the goldfish 22. Since the turtle has the most points, the turtle wins.

SCORE RATIONALE
The student described a method that meets the two criteria and explained how the turtle would win using this method. This response provides evidence of clear and effective reasoning about the situation, and earns full credit.

Student Sample B

1st choice = 2 points
2nd choice = 1 point
Turtle: \(18 + 5 = 23\)
Hamster: \(8 + 7 = 15\)
Goldfish: \(14 + 8 = 22\)

A point system with the first vote being 2 points and the second vote being 1 point would make the turtle win because it would have the most points. Also, if both votes were the same the goldfish would win because it had more points that the goldfish or the hamster.

SCORE RATIONALE
This response includes a method that meets the two criteria and an explanation of how the turtle would win using this method. The student provided additional information about an alternative method that fits the criteria but would have a different resulting winner. The response contains evidence of the student’s reasoning and modeling to the full extent expected by this item.
Student Sample C

1st = 10 p  
2nd = 1 p  

T 90 + 5 = 95  
H 40 + 7 = 47  
G 70 + 8 = 78  

I decided that the 1st vote would cost 10 points and the 2nd vote 1 point. The turtle got 95 points in total, the hamster got 47 points in total, and the goldfish got 78 points in total. Which means the turtle won for the overall class pet.

SCORE RATIONALE
This response describes a method that meets the two criteria and explains how the turtle would win using this method. The response provides clear evidence of the student's reasoning and modeling to the full extent expected by this item.

Student Sample D

If each vote were worth 1 point, the goldfish would have 15 points, the hamster would have 11 points, and the turtle would have 16 points, which means 16 is greater than 15 and 16 is greater than 11.

SCORE RATIONALE
Although the numbers used are based on an incorrect response to item 1, this response describes a method that meets the two criteria and explains how the turtle would win using this method. The response contains evidence of the student's competence in problem solving and reasoning and modeling to the full extent expected by this item.
Student Sample E

The first choice point equals 5 points
Turtle: 7 x 5 = 35
Goldfish: 4 x 5 = 20
Hamster: 9 x 5 = 45

The second choice point is worth 3
Turtle: 9 x 3 = 27
Goldfish: 8 x 3 = 24
Hamster = 4 x 3 = 12

Turtle: 35 + 27 = 62
Goldfish: 20 + 24 = 44
Hamster 45 + 12 = 57

SCORE RATIONALE
The student described a method that meets the two criteria: 1st choice is worth 5 points and 2nd choice worth 3 points. However, the method was incorrectly applied to the data from item 1, and no explanation is provided to support the turtle winning. This student has demonstrated only a partial understanding of the mathematical content and practices essential to this item.
### Student Sample F

<table>
<thead>
<tr>
<th>Animal</th>
<th>1st Choice</th>
<th>2nd Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hampster</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>Turtles</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>Goldfish</td>
<td>7</td>
<td>16</td>
</tr>
</tbody>
</table>

**SCORE RATIONALE**
The response presents a method that meets the two criteria but is incorrectly applied to the data. Both 1st and 2nd choice votes for the turtle were doubled. Only 2nd choice votes were doubled for the hamster and goldfish. Due to the incorrect application of the method, the explanation for the turtle winning isn’t valid. This response demonstrates a partial understanding of the essential processes it assesses.

### Student Sample G

<table>
<thead>
<tr>
<th>Animal</th>
<th>1st Choice</th>
<th>2nd Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamster</td>
<td>3 points</td>
<td>1 point</td>
</tr>
<tr>
<td>Goldfish</td>
<td>12 points</td>
<td>7 points</td>
</tr>
<tr>
<td>Turtle</td>
<td>27 points</td>
<td>5 points</td>
</tr>
</tbody>
</table>

**SCORE RATIONALE**
This response partially meets the requirements of the problem. The student presented a method that meets the two criteria, but the response does not include an explanation or make clear why the turtle would win using this method. The response earns 1 point.
Student Sample H

Well, the students all had the choice to pick what pet they wanted they all picked 1st choices and 2nd choices. In total there are 20 students. $20 \times 2 = 40$. In total there are 40 votes because all the students had 2 votes. The turtle won because most of the students voted for the turtle as the class pet.

1st choice turtle: 9
2nd choice: 5

SCORE RATIONALE
This response does not provide a method that meets the two criteria required. While the student does explain why the turtle won, it is unrelated to the prompt and does not indicate a clear comprehension of the mathematical content essential to this problem. There is minimal evidence present to demonstrate relevant reasoning and/or modeling, and this response earns 0 points.

Student Sample I

To make the turtle win you can add two or up to either 1st choice or 2nd choice. For example,

Turtle at 1st = 14 points in all
14 + 2 = 16 votes

Reason: the greatest is 15 so that’s why I added two

SCORE RATIONALE
This student provided a method that allowed the turtle to win (giving him two more points) but it did not meet the design constraints provided in the prompt. This response suggests a misunderstanding of what the prompt is asking for, and there is minimal evidence present to demonstrate the relevant reasoning and/or modeling related to this problem. The response earns 0 points.
### Student Sample J

**0 POINTS**

So for each name or hamster that’s how many there would be so there is more turtle than others so the turtle won.

Ex:
- T = turtle
- G = goldfish
- For each letter = 1

<table>
<thead>
<tr>
<th>T</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>h</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>g</td>
</tr>
</tbody>
</table>

**SCORE RATIONALE**

In this response, the student presented data to show how the turtle could win, however he/she did not develop a method that met the two criteria given in the problem. As the explanation for the turtle winning was not based on a method meeting the requirements, it does not demonstrate understanding of the problem solving and modeling assessed by this problem. The response earns 0 points.

### Student Sample K

**0 POINTS**

3 = frist vote
3 = second vote

**SCORE RATIONALE**

The student described a method that meets the two criteria, however, the response does not include an explanation about why the turtle would win using this method. This response only partially demonstrates the reasoning and modeling being assessed by this problem.