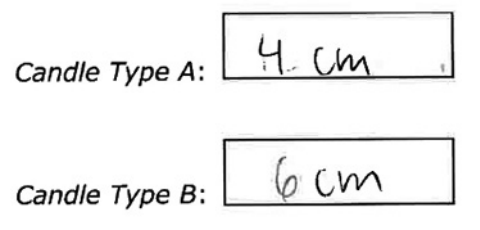
High School / Case Study 1

MATH * SMARTER BALANCED PERFORMANCE TASK

ENGLISH LEARNER

Item 1	Candles A and B are lit at the same time. What will be the height, in cm, of each candle after 3 hours of burning?
Student Response to Item 1	
Analysis of Response to Item 1	The response receives a score of 0 points. The student calculated the amount of candle burned in 4 hours for each candle instead of 3 hours.

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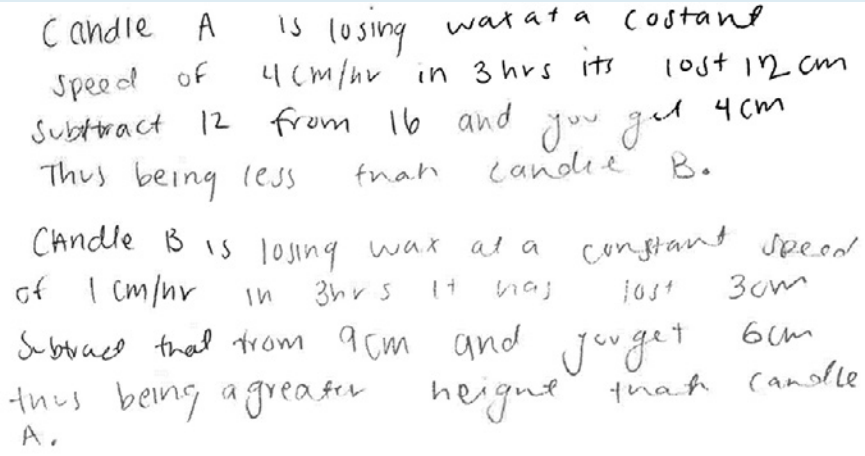
Item 2

Candles of each type were lit at the same time. Abbie thinks that since Candle Type A burns more quickly than Candle Type B, that it will burn out (have a height of 0 cm) first.

Julie thinks that since Candle Type B starts out much shorter than Candle Type A, it will be the candle to burn out first.

Which candle will burn out first? Give a mathematical explanation to convince Abbie and Julie of your solution. Clearly identify the quantities involved.

Student Response to Item 2


Candle A is losing wax at a costant speed of 4cm/hr in 3hrs its lost 12cm subtract 12 from 16 and you get 4cm Thus being less than candle B.

Candle B is losing wax at a constant speed of 1cm/hr in 3hrs it has lost 3cm Subtract that from 9cm and you get 6cm thus being a greater height than candle A.

Analysis of Response to Item 2

This response receives a full score of 2 points. The student correctly determined that Candle A will burn out first by comparing the burn rate of each candle: Candle A is losing 4 centimeters/hour and in 3 hours will lose 12 centimeters. Candle B is losing 1 centimeter/hour and in 3 hours will lose 3 centimeters. What the student used as an initial height is the height of each candle 1 hour after it is lit—for Candle A at 16 centimeters and Candle B at 9 centimeters.

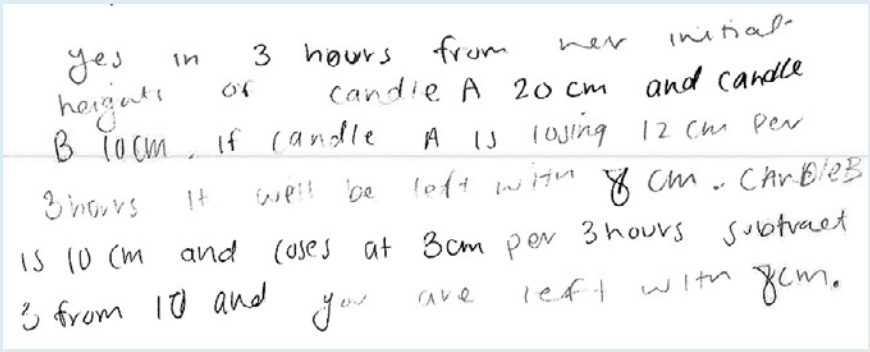
Item 3

Abbie has 3 hours left to film. She lights a **new** Candle Type A and Candle Type B and then starts filming.

In the 3 hours she has left, will Abbie capture the moment when the candles are exactly the same height?

Explain to Abbie how you can determine the answer.

Student Response to Item 3



Analysis of Response to Item 3

This response receives a partial score of 1 point. The student correctly reasoned from an incorrect calculation. The student's subtraction error prompted him/her to determine that in 3 hours, the height of Candle A is 8 centimeters (from 20 – 12 = 8), the same height as Candle B (from 10 – 3 = 8).

Item 4

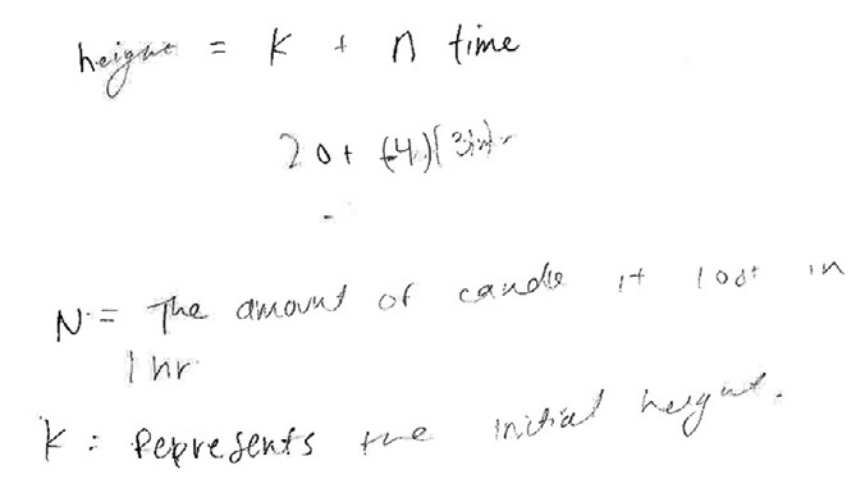
You have decided to use functions to help Abbie think about the candles.

You show her how to represent the height of a candle, ***h***, as a function of time, ***t***, using this equation:

$$h = k + nt$$

First, explain to Abbie what **k** and **n** represent in order to model the different candles. Be specific in your explanation.

Student Response to Item 4



Analysis of Response to Item 4

This response receives a full score of 2 points. The student correctly identified what n and k each represent in the equation $h = k + nt$.

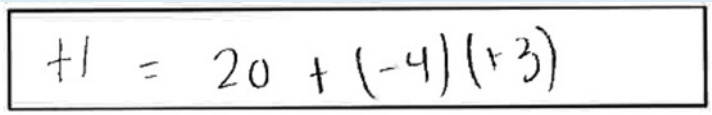
Item 5

Now, choose either Candle A or Candle B to create an equation that will tell Abbie the height of the candle at ***t*** hours after it is lit.

Determine what the numerical values for ***k*** and ***n*** should be for the candle you chose.

Using these ***k*** and ***n*** values, write an equation that tells Abbie the height h of the candle, in cm, at ***t*** hours after it is lit.

Student Response to Item 5



Analysis of Response to Item 5

This response receives a score of 0 points. The student did not generalize *h*, the height of either candle, in terms of *t*, the number of hours after the candle is lit.

Item 6

For her next film, Abbie wants candles that will burn for exactly 8 hours. You want to give her a choice by designing two different candles (Type C and Type D).

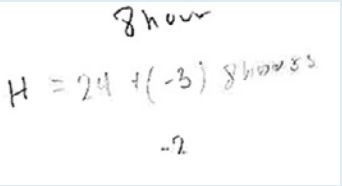
Using the equation ***h* = k + n*t*,** determine two different pairs of values for k and n that will meet the requirement to burn down to a height of 0 cm in exactly 8 hours.

Complete the table to show two possible sets of values for **k** and **n** for your new candle designs.

	k	n
Candle Type C		
Candle Type D		

Student Response to Item 6

	k	n
Candle Type C	24	-3
Candle Type D	16	-2



Analysis of Response to Item 6

This response receives a full score of 2 points. The student created values for k and n that resulted in Candle C and Candle D each burning out in 8 hours.

Overview of Student's Performance

This student, an emerging bilingual student, demonstrated a strong understanding of the mathematical content and practices essential to this task. From the work in item 1, the student seems to make sense of the problem (MP 1) by anchoring his/her thinking in the formulas $S = d/t$ and $d = S \cdot t$, where S = speed, d = distance, and t = time. The student tried to relate what s/he learned from $d = S \cdot t$ to finding the height, in centimeters, 3 hours after each candle is lit. The response shows a connection between the ratio of 4 centimeters/hour in 3 hours and the result of 12 centimeters of burned wax. S/he identified the burn rate for each candle, but miscalculated the height of each candle by finding the height 4 hours after each is lit, instead of 3 hours. His/her answer in item 2 is consistent with his/her thinking from item 1. Since s/he knew the burn rate for each candle, s/he was able to apply the pattern of constant decrease to determine the height of each candle after t hours to compare and identify which candle burns out first. S/he continually evaluated the reasonableness of intermediate results (MPs 1 and 8) by comparing the heights 4 hours after the candles are lit.

In item 3, the student determined that the heights for both candles are the same after 3 hours of burning. S/he tried to make sense of quantities in the given situation (MP 2), yet did not correctly compare the candles' heights after 3 hours due to a minor miscalculation ($10 - 3 = 8$).

For items 4, 5, and 6, the student made sense of the equation $h = k + nt$ (MP 1) by contextualizing the symbols and related h = height, k = initial height, n = amount of candle lost in 1 hour (burn rate), and t = time (MP 2). Because of his/her understanding of what each variable represents, the student was able to use the same structure to answer item 6 (MP 7). As s/he decoded each variable in the equation $h = k + nt$, s/he was able to analyze how Candle C and Candle D will burn down to a height of 0 cm in 8 hours. The student provided values k = 24 and n = -3, and k = 16 and n = -2. In item #5, the response does not earn full points because the student did not generalize the height, h, in terms of time, t, in hours. For this item, the student gave a specific example to answer the prompt: $h = 20 - (-4)(3)$. S/he seems to have focused on the second statement of the prompt, and the first part of the third statement, not understanding that the answer is supposed to be an equation that expresses h in terms of t.

Next Steps

This student has a partial understanding of the concepts this task is assessing. S/he would benefit from opportunities to engage in MPs 2 and 4, moving from real-world situations to mathematical representations of the quantitative relationships in the situations given, and vice versa. The student would also benefit from collaborative conversations with peers to gain more experience communicating about his/her understanding of the mathematical content. The student's approach to problem solving can also be supported with instruction guided by UDL Principle 1 (Provide multiple means of engagement) and UDL Principle 2 (Provide multiple means of representation). Some instructional strategies that can support his/her engagement with the mathematics are: (1) the Read-Draw-Write strategy, where the student is asked to read a problem once or twice (or more depending on his/her level of comprehension), draw a figure to visualize a problem, and write an expression, an equation, or an initial solution to make sense of the problem; (2) creating, analyzing, or presenting mathematical models, where the student is given many experiences to construct or use mathematical models to show his/her understanding of a quantity, an expression, or a situation; (3) use of multiple representations such as tables of values, diagrams, ordered pairs, verbal descriptions, and graphs, which can provide means for the student's comprehension and language development; (4) use of questioning and giving purposeful feedback to guide the student in using prior knowledge to connect to the current task; and (5) use of vocabulary development tools, such as concept maps, number webs, word walls, and/or Frayer Models (graphic organizers) to support the student's use of mathematical language for comprehension and problem solving. Providing opportunities for the student to learn different ways of approaching content and providing options for paths to make sense of a problem will have a huge impact on his/her mathematics learning.