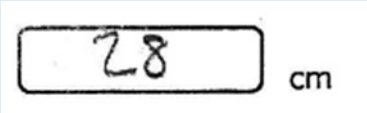


Grade 8 / Case Study 1

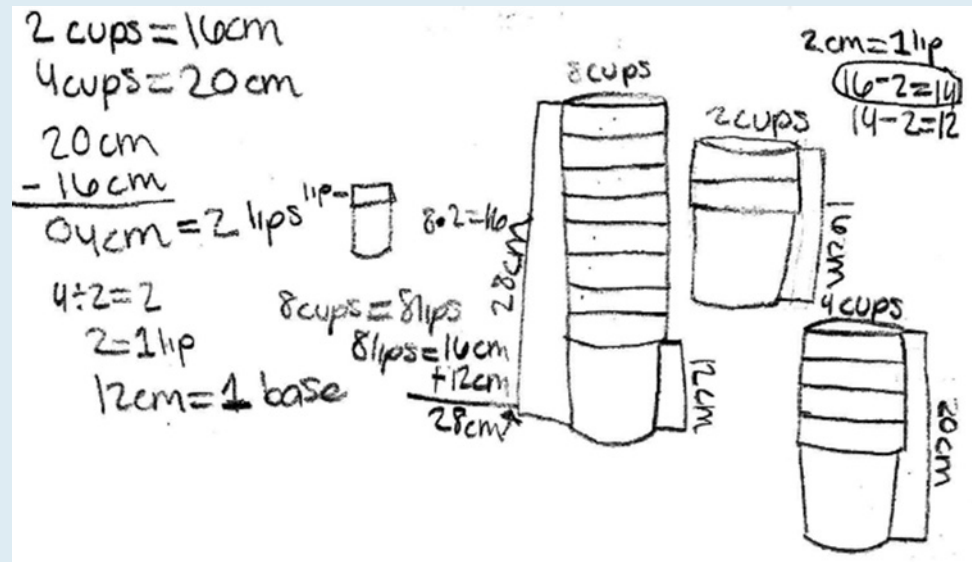
➔ MATH * SMARTER BALANCED PERFORMANCE TASK

Item 1	How tall, in cm, is the stack of 8 cups?
Student Response to Item 1	 A handwritten student response showing the number '28' inside a rectangular box, followed by the unit 'cm'.
Analysis of Response to Item 1	The response receives a full credit score of 1 point. The student gave the correct answer of 28.

Item 2

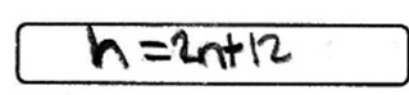
How tall, in cm, is 1 cup? Explain how you determined the height of 1 cup.

Student Response to Item 2



Analysis of Response to Item 2

This response receives a full credit score of 2 points. The student correctly calculated the height of one cup and provided a mathematically logical explanation as to how s/he calculated the height. The student found the difference between stack one and stack two, divided the difference between the stack heights by 2, and found that each "lip" was 2 centimeters. In the above diagrams, s/he stated "2 cm = 1 lip" and computed $16 - 2 = 14$. S/he circled this computation to indicate the answer.

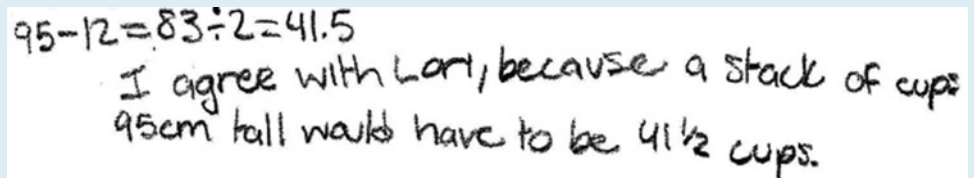
Item 3	<p>Your teacher thinks that instead of having to figure out these stacks each time, it would be useful to understand the general relationship.</p> <p>Write an equation expressing the relationship between the height of the stack and the number of cups in the stack.</p> <p>Let h represent the height of the stack, in cm, and n the number of cups in the stack.</p>
Student Response to Item 3	 A handwritten equation $h = 2n + 12$ is shown inside a rectangular box. The handwriting is in black ink on a white background.
Analysis of Response to Item 3	<p>This response receives a full credit score of 1 point. The student wrote a correct equation based on the calculations shown in the previous problem, where s/he determined a height of $14 - 2$ (another lip) = 12 centimeters for the base and a rate of growth of 2.</p>

Item 4

The catalog is advertising a stack of these cups that is 95 cm tall. Lori says, "That must be a misprint because a stack of that height is not possible."

Do you agree or disagree with Lori? Explain your reasoning.

Student Response to Item 4



$95 - 12 = 83 \div 2 = 41.5$
I agree with Lori, because a stack of cups 95cm tall would have to be $41\frac{1}{2}$ cups.

Analysis of Response to Item 4

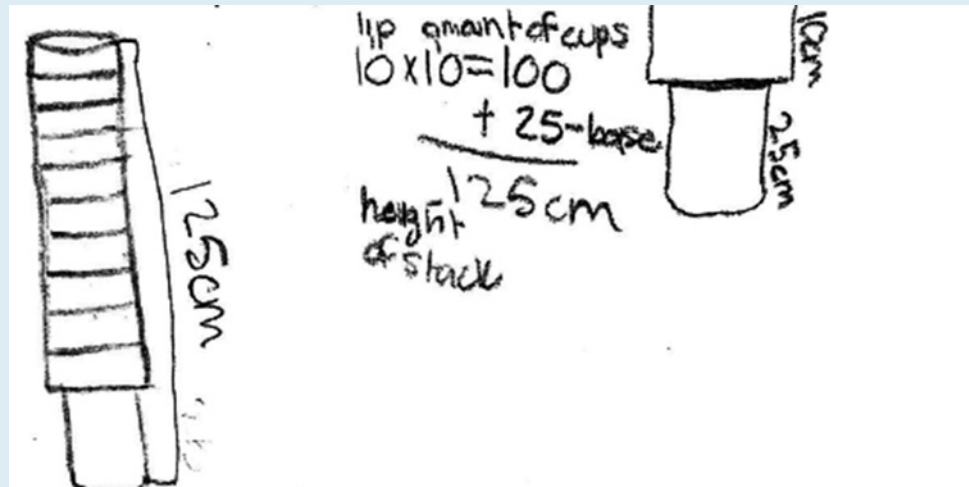
This response receives a full credit score of 2 points. The student agreed with Lori and provided a valid mathematical explanation as to why a stack of cups could not reach 95 centimeters. S/he worked backward in an equation using relevant values and arrived at a non-whole number, 41.5. Although there is a clear misuse of the equal sign in the extended equation, this had no impact on the score since this item is assessing communication of reasoning, not mathematical precision or an ability to set up an equation.

Item 5

Your teacher wants to sell School Spirit Cups with your school logo on them. She asks you to design this new cup such that a stack of 10 of them will be 125 cm tall.

Describe key measurements of the School Spirit Cups and explain how they will meet your teacher's specifications.

Student Response to Item 5



Analysis of Response to Item 5

This response receives a full credit score of 2 points. The student drew a picture showing the key measurements and then showed calculations in which 10 cup lips of 10 centimeters is equal to 100 centimeters, which added to the base of 25 centimeters gives a stack height of 125 centimeters. The mathematical calculation with labels makes this an adequate response.

Overview of Student's Performance

This student's responses show a strong understanding of the task throughout each of the items in the problem set. S/he determined that the stack of 8 cups was 28 centimeters tall. The work presented in item 2 provides insight into how s/he reached the answers for items 1–3. Use of pictures and equations based on those pictures makes the responses thoroughly developed. This work indicates the use of Mathematical Practice (MP) 8. By applying repeated reasoning to compare the first two stacks, the student was able to determine the height of the third stack. The work shows the development of the base and the growth pattern of the lips, which led to the correct equation of $h = 2n + 12$.

The response to item 2 also indicates that this student is resourceful and has likely had opportunities to look for and interpret patterns. Having no vocabulary for the parts of the cup, the student developed a way to communicate the parts of the cup that were crucial to understanding and solving the problems. As a mathematically proficient student, s/he maintained oversight of the process while attending to the details. S/he continually evaluated the reasonableness of his/her intermediate results (MP 1), was very thorough, and labeled and supported his/her steps with reasoning (MPs 3 and 6).

For item 4, the student used the information given along with the equation developed in item 3 and agreed with Lori, because his/her calculation showed that 95 centimeters could only be reached with a fractional number of cups. Because of previous work, item 5 was easily completed by drawing the target diagram and seeing the number relationship that 10 lips of 10 cups are 100 centimeters tall, leaving 25 centimeters for the base. The calculations support the diagram, as does the drawing of a single cup with a base of 25 centimeters and a lip of 10 centimeters to show the complete cup. This response demonstrates the student's clear attention to precision of language as well as appropriate use of symbols and defining variables by labeling the lip and base.

Next Steps

This student would benefit from support in understanding the syntax of the sentences containing multiple equal signs. Students often construct sentences such as this in elementary school and carry the structure on through to algebraic sentences. The first line in the response to item 4 has a series of calculations connected by equal signs. This is a common error at this grade level. The student would benefit from opportunities that build greater attention to precision (MP 6) when representing calculations with equations. Asking the student if $95 - 12 = 83 \div 2$ would help him/her reason this through.

This task involved several opportunities to demonstrate MP 3. Although the student provided correct responses, s/he could benefit from reading through responses and ensuring that his/her reasoning is complete and that the reader does not need to infer the conclusion. For example, while the response to item 4 is correct, stating explicitly that a half of a cup is not reasonable in this situation would have strengthened the student's response.

Finally, this student would benefit from opportunities to translate between visual representations and written explanations. The student's labeled pictures are a solid ground to work from, but because the assessment system's interface allows only text input for many items, it is important for this student to be accustomed to producing complete written arguments and explanations to communicate ideas that can also be represented visually.